

Supplementary material: A certified Branch & Bound approach for Reliability-Based Optimization problems, submitted to *Journal of Global Optimization*

Benjamin Martin ^{*1,2}, Marco Correia ^{†1}, and Jorge Cruz ^{‡1}

¹NOVA-LINCS, Universidade Nova de Lisboa

²LIX, Ecole Polytechnique

This document contains the detailed experimental results on the interval Branch & Bound for Reliability-based optimization problems presented in: *A certified Branch & Bound approach for Reliability-Based Optimization problems*; from the same authors submitted to Journal of Global Optimization. See therein for details about the implementation. This document is provided with an archive containing all the raw data, tables and figures sources¹.

Notations

f_L : Global lower bound,

f_U : Global upper bound,

$\text{wid}(\mathbf{f})$: $f_U - f_L$

t : CPU time (s). Does not count initialization (negligible),

t_{UB} : CPU time (s) taken by the quadrature applied to upper bound candidates,

^{*}bmartin@lix.polytechnique.fr

[†]mvc@fct.unl.pt

[‡]jerc@fct.unl.pt

¹<http://ben-martin.fr/files/publications/materials/RBO/dataRBO.tar.gz>

t_{ups} : CPU time (s) for updating the sharing structure (i.e. transfer the random box from the father node to the child),
 t_{box} : CPU time (s) taking by the quadrature applied to decision box,
 $wid(\mathbf{f}_r)$: $(f_U - f_L) / \max(|f_U|, 1)$,
 $|\mathcal{S}|$: Number of nodes in $\mathcal{S}$,
 $|\mathcal{S}_{out}|$: Number of nodes in $\mathcal{S}_{out}$,
 $|\mathcal{S}_T|$: $|\mathcal{S}| + |\mathcal{S}_{out}|$,
 $\max |\mathcal{S}_T|$: Maximum number of $|\mathcal{S}_T|$,
 $|\mathcal{H}_{\square}|$: Number of shared random boxes (for quadrature over decision boxes),
 $\max |\mathcal{H}_{\square}|$: Maximum number of random boxes (for quadrature over decision boxes).

Note that each raw data file contains the results at different iterations of the Branch & Bound. Each line correspond to an iteration, each column to a measure corresponding to the one in the description above, in that order (e.g. the fourth column corresponds to t). The raw data files themselves are separated into two directories, one for each variant of the Branch & Bound. Considering n problems and m possible K^{box} values and 2 possible quadrature functions (default or linear-model based), the files are numbered from 0 to $2nm - 1$. They are organised as follows:

- the results for problem j are stored between files $(j - 1)2m$ to $(j - 1)2m + 2m - 1$. E.g, $n = 10, m = 5$, results for problem 5 are stored into the files from 40 to 49.
- for an even i , files i and $i + 1$ correspond to the same algorithm and problem respectively without or with linear-model based quadrature.
- for a problem j and induced data files, each two files (e.g. i and $i + 2$) correspond to the same algorithm with an increased K^{box} .

Results

The experimental results are presented problem by problem as described in Table 1. The shared and non-shared version of the Branch & Bound are presented in separate tables/figures. For the figures, the level of gray represent

the value of K^{box} (we recall, $K^{box} \in \{500, 1000, 2000, 5000, 10000, 20000\}$ for *non-shared* Branch & Bound, $K^{box} \in \{20, 50, 100, 200, 500\}$ for the *shared* one): the higher the level of gray, the higher is K^{box} . Plain lines and dashed lines correspond respectively to the default reliability evaluation or the linear model-based.

Table 1: Problem details

Problem	1	2	3	4	5	6	7	8	9	10
Instance	RBO1	RBO1	RBO2	RBO2	RBO3	RBO3	RBO4	RBO4	RBO5	RBO5
r	0.9	0.99	0.9	0.99	0.9	0.99	0.9	0.99	0.9	0.99

Problem 1

Table 2: Results for *non-shared* variant on Problem 1

Method	t	$\text{wid}(\mathbf{f}_r)$	$ \mathcal{S}_T $	$\max \mathcal{S}_T $	$ \mathcal{H}_\square $	$\max \mathcal{H}_\square $
(500, False)	77.35	0.0236	1 511	1 512	0	501
(500, True)	4.98	0.0089	17	17	0	501
(1 000, False)	19.35	0.0089	31	72	0	1 001
(1 000, True)	6.16	0.0089	16	16	0	1 001
(2 000, False)	10.40	0.0089	27	30	0	2 001
(2 000, True)	12.46	0.0089	16	16	0	2 001
(5 000, False)	14.08	0.0089	24	24	0	5 001
(5 000, True)	24.95	0.0089	16	16	0	5 001
(10 000, False)	22.43	0.0089	23	23	0	10 001
(10 000, True)	43.46	0.0089	16	16	0	10 001
(20 000, False)	41.56	0.0089	23	23	0	20 001
(20 000, True)	88.04	0.0089	16	16	0	20 001

Table 3: Results for *shared* variant on Problem 1

Method	t	$\text{wid}(\mathbf{f}_r)$	$ \mathcal{S}_T $	$\max \mathcal{S}_T $	$ \mathcal{H}_\square $	$\max \mathcal{H}_\square $
(20, False)	425.99	0.0241	2 152	2 945	880 953	1 206 187
(20, True)	1.51	0.0089	18	18	4 327	4 327
(50, False)	71.09	0.0089	54	576	30 599	540 956
(50, True)	2.17	0.0089	17	17	10 404	10 404
(100, False)	5.92	0.0089	39	82	44 303	109 726
(100, True)	3.35	0.0089	16	16	19 401	19 401
(200, False)	5.06	0.0089	31	44	70 459	104 323
(200, True)	5.33	0.0089	16	16	38 680	38 680
(500, False)	7.84	0.0089	26	27	148 457	148 457
(500, True)	11.17	0.0089	16	16	96 213	96 213

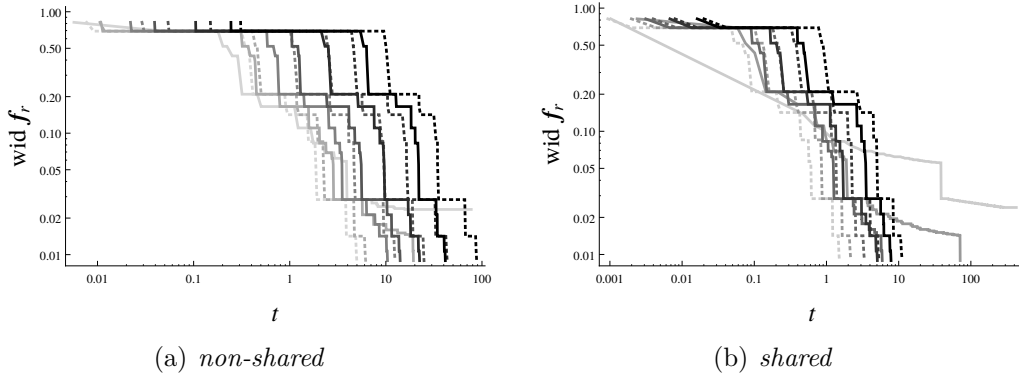


Figure 1: Problem 1: t VS $\text{wid}(\mathbf{f}_r)$

Problem 2

Table 4: Results for *non-shared* variant on Problem 2

Method	t	$\text{wid}(\mathbf{f}_r)$	$ \mathcal{S}_T $	$\max \mathcal{S}_T $	$ \mathcal{H}_\square $	$\max \mathcal{H}_\square $
(500, False)	90.53	0.0191	1 493	1 493	0	501
(500, True)	4.84	0.0063	9	17	0	501
(1 000, False)	12.71	0.0063	21	47	0	1 001
(1 000, True)	6.70	0.0063	9	14	0	1 001
(2 000, False)	9.85	0.0063	17	34	0	2 001
(2 000, True)	10.70	0.0063	9	13	0	2 001
(5 000, False)	16.39	0.0063	14	26	0	5 001
(5 000, True)	21.82	0.0063	8	12	0	5 001
(10 000, False)	25.15	0.0063	14	24	0	10 001
(10 000, True)	38.51	0.0063	8	12	0	10 001
(20 000, False)	39.76	0.0063	12	22	0	20 001
(20 000, True)	73.27	0.0063	8	12	0	20 001

Table 5: Results for *shared* variant on Problem 2

Method	t	$\text{wid}(\mathbf{f}_r)$	$ \mathcal{S}_T $	$\max \mathcal{S}_T $	$ \mathcal{H}_\square $	$\max \mathcal{H}_\square $
(20, False)	662.29	0.0185	3 103	3 929	1 408 063	1 770 698
(20, True)	2.12	0.0063	13	23	3 106	5 381
(50, False)	17.49	0.0063	46	203	25 973	182 910
(50, True)	2.40	0.0063	10	18	6 045	10 124
(100, False)	5.26	0.0063	30	74	34 379	95 025
(100, True)	2.74	0.0063	9	15	10 819	16 907
(200, False)	5.40	0.0063	23	48	53 220	113 819
(200, True)	4.11	0.0063	9	13	21 608	29 514
(500, False)	7.99	0.0063	18	33	104 781	189 575
(500, True)	8.27	0.0063	8	12	48 147	67 936

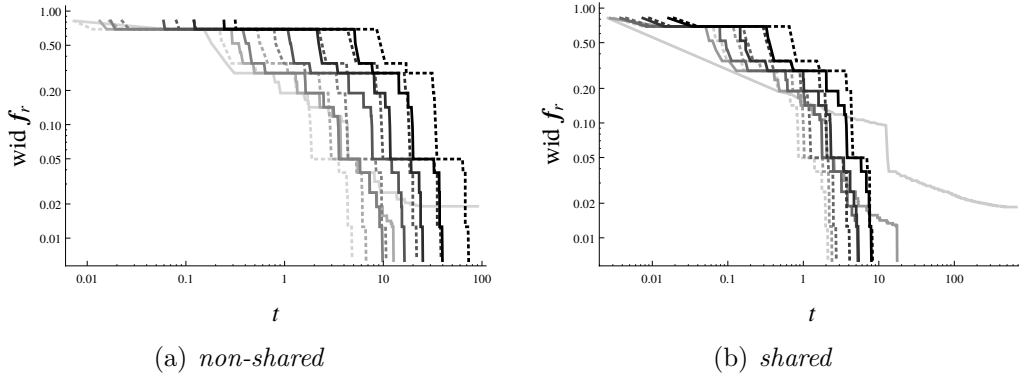


Figure 2: Problem 2: t VS $\text{wid}(\mathbf{f}_r)$

Problem 3

Table 6: Results for *non-shared* variant on Problem 3

Method	t	$\text{wid}(\mathbf{f}_r)$	$ \mathcal{S}_T $	$\max \mathcal{S}_T $	$ \mathcal{H}_\square $	$\max \mathcal{H}_\square $
(500, False)	56.96	0.0204	1 144	1 144	0	501
(500, True)	5.38	0.0076	25	25	0	501
(1 000, False)	106.59	0.0105	1 052	1 052	0	1 001
(1 000, True)	7.58	0.0076	25	25	0	1 001
(2 000, False)	6.95	0.0076	37	37	0	2 001
(2 000, True)	13.59	0.0076	23	25	0	2 001
(5 000, False)	11.35	0.0076	30	30	0	5 001
(5 000, True)	27.89	0.0076	22	22	0	5 001
(10 000, False)	18.18	0.0076	28	28	0	10 001
(10 000, True)	50.07	0.0076	22	22	0	10 001
(20 000, False)	30.97	0.0076	26	26	0	20 001
(20 000, True)	90.41	0.0076	22	22	0	20 001

Table 7: Results for *shared* variant on Problem 3

Method	t	$\text{wid}(\mathbf{f}_r)$	$ \mathcal{S}_T $	$\max \mathcal{S}_T $	$ \mathcal{H}_\square $	$\max \mathcal{H}_\square $
(20, False)	3600.00	0.0142	7 968	9 083	4 456 173	4 982 901
(20, True)	1.59	0.0076	30	30	8 129	8 129
(50, False)	26.56	0.0076	64	388	44 706	424 926
(50, True)	2.29	0.0076	25	25	17 238	17 238
(100, False)	3.39	0.0076	48	48	66 153	75 822
(100, True)	3.48	0.0076	24	25	33 464	34 200
(200, False)	3.29	0.0076	40	40	110 740	110 740
(200, True)	5.47	0.0076	23	25	64 462	68 848
(500, False)	4.93	0.0076	32	32	221 068	221 068
(500, True)	10.31	0.0076	22	22	154 225	154 225

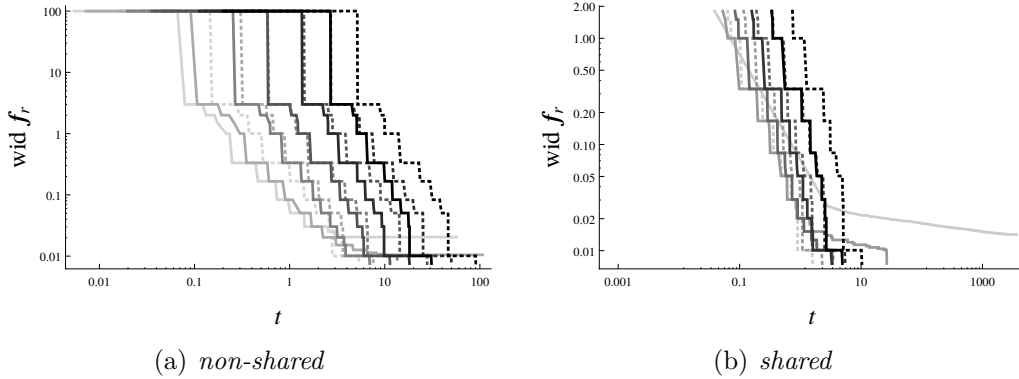


Figure 3: Problem 3: t VS $\text{wid}(\mathbf{f}_r)$

Problem 4

Table 8: Results for *non-shared* variant on Problem 4

Method	t	$\text{wid}(\mathbf{f}_r)$	$ \mathcal{S}_T $	$\max \mathcal{S}_T $	$ \mathcal{H}_{\square} $	$\max \mathcal{H}_{\square} $
(500, False)	58.55	0.0602	1 052	1 052	0	501
(500, True)	6.23	0.0070	7	21	0	501
(1 000, False)	122.15	0.0395	1 174	1 174	0	1 001
(1 000, True)	6.61	0.0070	7	16	0	1 001
(2 000, False)	264.64	0.0308	1 250	1 250	0	2 001
(2 000, True)	8.88	0.0070	6	13	0	2 001
(5 000, False)	12.08	0.0070	7	21	0	5 001
(5 000, True)	14.85	0.0070	6	12	0	5 001
(10 000, False)	15.20	0.0070	7	18	0	10 001
(10 000, True)	28.48	0.0070	6	12	0	10 001
(20 000, False)	23.60	0.0070	6	15	0	20 001
(20 000, True)	54.43	0.0070	6	12	0	20 001

Table 9: Results for *shared* variant on Problem 4

Method	t	$\text{wid}(\mathbf{f}_r)$	$ \mathcal{S}_T $	$\max \mathcal{S}_T $	$ \mathcal{H}_{\square} $	$\max \mathcal{H}_{\square} $
(20, False)	3600.00	0.0455	5 603	9 236	3 059 381	5 183 369
(20, True)	2.45	0.0070	9	31	2 007	9 904
(50, False)	3600.02	0.0318	3 291	5 270	4 631 517	7 472 345
(50, True)	3.00	0.0070	7	18	4 127	13 646
(100, False)	1276.50	0.0070	10	1 908	11 480	5 408 945
(100, True)	2.60	0.0070	7	15	8 334	21 021
(200, False)	11.34	0.0070	9	93	20 644	401 890
(200, True)	3.37	0.0070	6	14	14 216	39 110
(500, False)	5.94	0.0070	8	30	45 206	256 892
(500, True)	5.87	0.0070	6	12	35 677	82 953

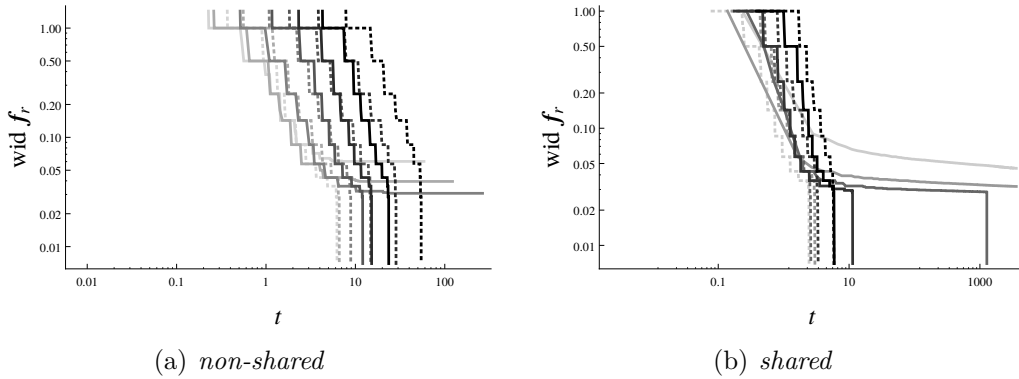


Figure 4: Problem 4: t VS $\text{wid}(\mathbf{f}_r)$

Problem 5

Table 10: Results for *non-shared* variant on Problem 5

Method	t	$\text{wid}(\mathbf{f}_r)$	$ \mathcal{S}_T $	$\max \mathcal{S}_T $	$ \mathcal{H}_\square $	$\max \mathcal{H}_\square $
(500, False)	84.00	0.0422	2 244	2 244	0	487
(500, True)	13.06	0.0099	43	43	0	494
(1 000, False)	193.17	0.0196	2 280	2 280	0	981
(1 000, True)	15.91	0.0099	38	39	0	987
(2 000, False)	66.90	0.0100	310	310	0	1 971
(2 000, True)	24.64	0.0097	32	32	0	1 978
(5 000, False)	21.16	0.0099	50	51	0	4 953
(5 000, True)	43.98	0.0097	29	29	0	4 965
(10 000, False)	22.19	0.0099	35	35	0	9 936
(10 000, True)	73.25	0.0097	29	29	0	9 955
(20 000, False)	37.79	0.0099	33	33	0	19 901
(20 000, True)	125.00	0.0097	29	29	0	19 937

Table 11: Results for *shared* variant on Problem 5

Method	t	$\text{wid}(\mathbf{f}_r)$	$ \mathcal{S}_T $	$\max \mathcal{S}_T $	$ \mathcal{H}_\square $	$\max \mathcal{H}_\square $
(20, False)	1054.19	0.0349	4 824	4 829	2 263 438	2 265 936
(20, True)	5.11	0.0099	54	55	16 727	17 042
(50, False)	1570.72	0.0147	3 925	3 927	4 533 217	4 535 475
(50, True)	7.06	0.0099	40	40	30 012	30 012
(100, False)	75.95	0.0100	648	653	1 217 067	1 226 909
(100, True)	9.57	0.0097	37	37	55 396	55 396
(200, False)	12.62	0.0100	142	142	442 876	442 876
(200, True)	11.92	0.0097	33	33	96 984	96 984
(500, False)	9.14	0.0099	64	64	465 261	465 261
(500, True)	20.14	0.0097	30	30	219 956	219 956

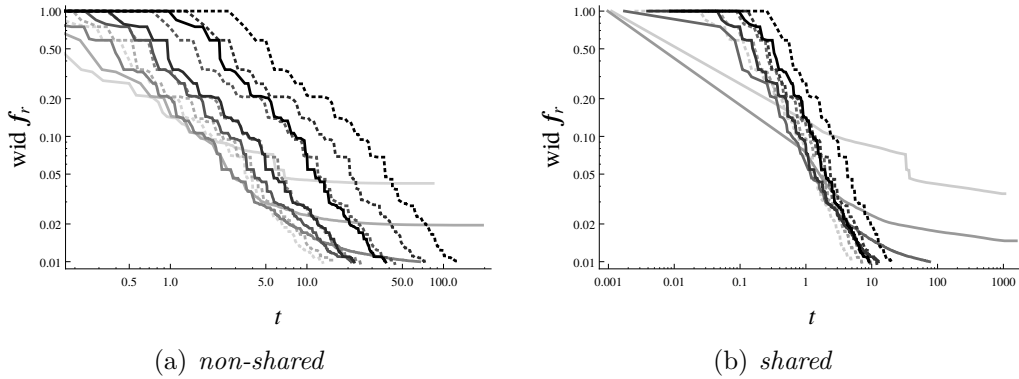


Figure 5: Problem 5: t VS $\text{wid}(\mathbf{f}_r)$

Problem 6

Table 12: Results for *non-shared* variant on Problem 6

Method	t	$\text{wid}(\mathbf{f}_r)$	$ \mathcal{S}_T $	$\max \mathcal{S}_T $	$ \mathcal{H}_\square $	$\max \mathcal{H}_\square $
(500, False)	135.19	0.1413	2 719	2 719	0	497
(500, True)	61.35	0.0100	178	178	0	501
(1 000, False)	234.10	0.0528	2 211	2 211	0	996
(1 000, True)	25.82	0.0100	44	46	0	997
(2 000, False)	531.53	0.0244	2 666	2 666	0	1 990
(2 000, True)	29.94	0.0100	36	36	0	1 996
(5 000, False)	135.07	0.0100	255	258	0	4 985
(5 000, True)	49.76	0.0100	33	33	0	4 990
(10 000, False)	60.76	0.0100	91	92	0	9 977
(10 000, True)	76.51	0.0100	32	32	0	9 984
(20 000, False)	68.79	0.0096	67	67	0	19 970
(20 000, True)	132.25	0.0100	32	32	0	19 978

Table 13: Results for *shared* variant on Problem 6

Method	t	$\text{wid}(\mathbf{f}_r)$	$ \mathcal{S}_T $	$\max \mathcal{S}_T $	$ \mathcal{H}_\square $	$\max \mathcal{H}_\square $
(20, False)	3600.00	0.1185	5 478	5 519	2 443 505	2 464 099
(20, True)	144.99	0.0100	520	529	197 389	201 200
(50, False)	3600.00	0.0355	3 837	3 983	4 101 753	4 269 267
(50, True)	11.44	0.0100	54	54	35 398	35 407
(100, False)	3600.02	0.0190	3 321	3 322	6 911 694	6 914 074
(100, True)	10.51	0.0100	36	37	44 363	46 147
(200, False)	360.44	0.0099	889	893	3 088 649	3 103 616
(200, True)	13.33	0.0100	36	36	87 317	87 317
(500, False)	25.63	0.0099	133	133	873 046	875 067
(500, True)	24.57	0.0100	33	33	198 104	198 104

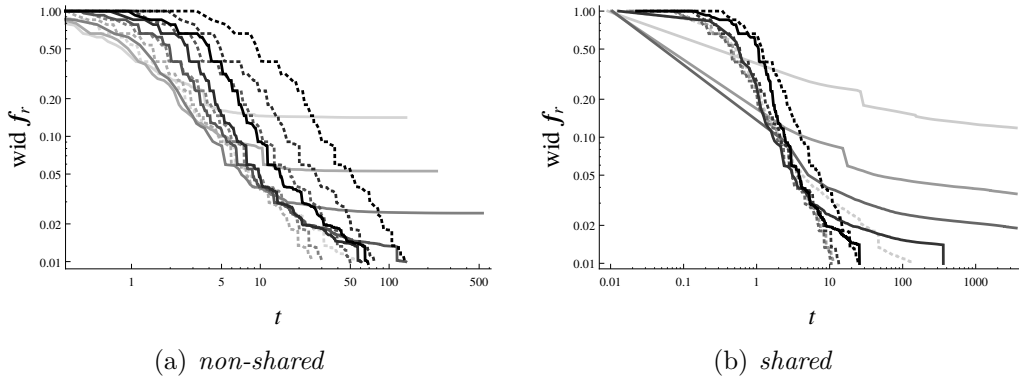


Figure 6: Problem 6: t VS $\text{wid}(\mathbf{f}_r)$

Problem 7

Table 14: Results for *non-shared* variant on Problem 7

Method	t	$\text{wid}(\mathbf{f}_r)$	$ \mathcal{S}_T $	$\max \mathcal{S}_T $	$ \mathcal{H}_\square $	$\max \mathcal{H}_\square $
(500, False)	69.64	0.6661	1 133	1 134	0	501
(500, True)	9.66	0.0088	6	30	0	501
(1 000, False)	504.93	0.6073	1 537	1 545	0	1 001
(1 000, True)	12.46	0.0088	6	29	0	1 001
(2 000, False)	827.73	0.0931	1 279	1 281	0	2 001
(2 000, True)	19.00	0.0088	6	26	0	2 001
(5 000, False)	1835.57	0.0534	1 225	1 236	0	5 001
(5 000, True)	33.94	0.0088	6	24	0	5 001
(10 000, False)	3153.44	0.0419	1 151	1 170	0	10 001
(10 000, True)	63.13	0.0088	6	23	0	10 001
(20 000, False)	3600.58	0.0365	242	293	0	20 001
(20 000, True)	116.35	0.0088	6	23	0	20 001

Table 15: Results for *shared* variant on Problem 7

Method	t	$\text{wid}(\mathbf{f}_r)$	$ \mathcal{S}_T $	$\max \mathcal{S}_T $	$ \mathcal{H}_\square $	$\max \mathcal{H}_\square $
(20, False)	3194.60	0.6461	2 360	3 345	1 112 719	1 604 095
(20, True)	8.75	0.0088	8	42	1 360	7 837
(50, False)	3433.18	0.5937	2 118	2 447	2 656 514	3 075 359
(50, True)	8.02	0.0088	7	31	2 839	13 666
(100, False)	3600.00	0.0834	152	1 181	296 575	2 700 252
(100, True)	14.28	0.0088	6	29	4 490	24 485
(200, False)	3600.03	0.0579	424	1 008	1 921 442	4 726 163
(200, True)	20.32	0.0088	6	26	8 966	43 055
(500, False)	3600.03	0.0431	729	747	8 721 110	8 778 244
(500, True)	30.37	0.0088	6	25	22 368	101 695

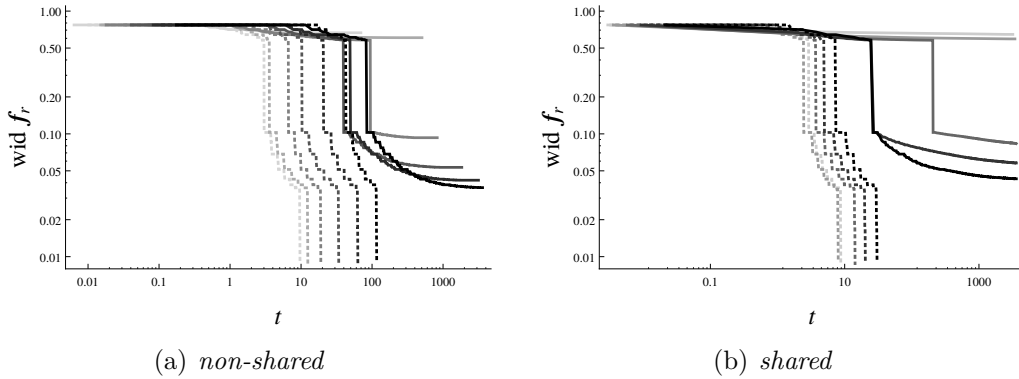


Figure 7: Problem 7: t VS $\text{wid}(\mathbf{f}_r)$

Problem 8

Table 16: Results for *non-shared* variant on Problem 8

Method	t	$\text{wid}(\mathbf{f}_r)$	$ \mathcal{S}_T $	$\max \mathcal{S}_T $	$ \mathcal{H}_\square $	$\max \mathcal{H}_\square $
(500, False)	97.88	0.6419	1 145	1 146	0	501
(500, True)	9.18	0.0067	8	30	0	501
(1 000, False)	508.46	0.5351	1 139	1 141	0	1 001
(1 000, True)	12.50	0.0067	8	28	0	1 001
(2 000, False)	1192.68	0.1050	1 290	1 300	0	2 001
(2 000, True)	17.56	0.0067	8	25	0	2 001
(5 000, False)	2471.63	0.0479	1 342	1 357	0	5 001
(5 000, True)	35.45	0.0067	8	22	0	5 001
(10 000, False)	3600.40	0.0338	969	969	0	10 001
(10 000, True)	66.52	0.0067	8	22	0	10 001
(20 000, False)	3600.80	0.0276	228	259	0	20 001
(20 000, True)	114.09	0.0067	7	21	0	20 001

Table 17: Results for *shared* variant on Problem 8

Method	t	$\text{wid}(\mathbf{f}_r)$	$ \mathcal{S}_T $	$\max \mathcal{S}_T $	$ \mathcal{H}_\square $	$\max \mathcal{H}_\square $
(20, False)	3523.78	0.6191	1 967	3 040	1 003 032	1 570 052
(20, True)	6.61	0.0067	10	39	2 490	6 588
(50, False)	3600.00	0.5167	717	1 168	896 061	1 446 617
(50, True)	7.88	0.0067	8	30	4 808	12 018
(100, False)	3600.01	0.1121	371	749	885 754	1 787 028
(100, True)	11.88	0.0067	8	28	9 608	22 078
(200, False)	3600.08	0.0610	388	667	1 828 676	3 198 789
(200, True)	14.45	0.0067	8	25	19 208	38 667
(500, False)	3600.06	0.0370	303	522	3 523 295	6 249 435
(500, True)	27.10	0.0067	8	22	47 999	83 122

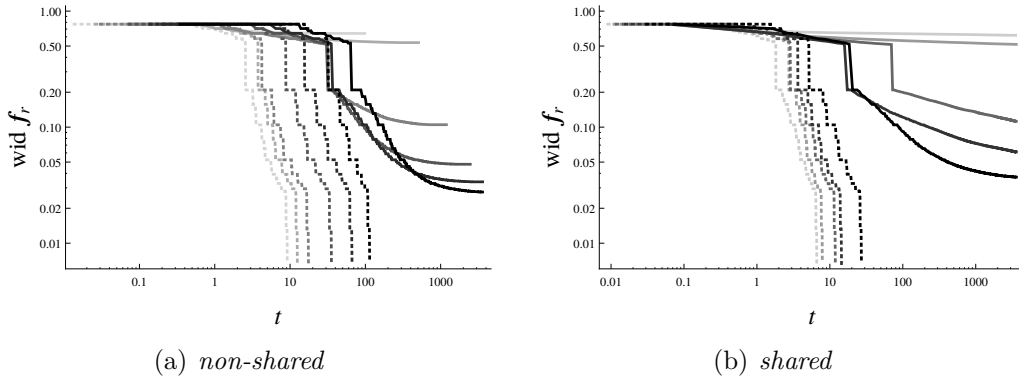


Figure 8: Problem 8: t VS $\text{wid}(\mathbf{f}_r)$

Problem 9

Table 18: Results for *non-shared* variant on Problem 9

Method	t	$\text{wid}(\mathbf{f}_r)$	$ \mathcal{S}_T $	$\max \mathcal{S}_T $	$ \mathcal{H}_\square $	$\max \mathcal{H}_\square $
(500, False)	103.15	0.0933	1 570	1 570	0	501
(500, True)	7.91	0.0092	12	15	0	501
(1 000, False)	315.99	0.0689	1 592	1 592	0	1 001
(1 000, True)	12.75	0.0092	11	15	0	1 001
(2 000, False)	916.11	0.0434	1 644	1 644	0	2 001
(2 000, True)	21.88	0.0092	11	15	0	2 001
(5 000, False)	2208.51	0.0252	1 789	1 789	0	5 001
(5 000, True)	48.40	0.0092	11	15	0	5 001
(10 000, False)	3600.12	0.0189	1 242	1 242	0	10 001
(10 000, True)	94.77	0.0092	11	15	0	10 001
(20 000, False)	1422.51	0.0092	235	235	0	20 001
(20 000, True)	182.66	0.0092	11	15	0	20 001

Table 19: Results for *shared* variant on Problem 9

Method	t	$\text{wid}(\mathbf{f}_r)$	$ \mathcal{S}_T $	$\max \mathcal{S}_T $	$ \mathcal{H}_\square $	$\max \mathcal{H}_\square $
(20, False)	414.14	0.0836	2 221	2 290	1 052 766	1 083 350
(20, True)	3.48	0.0092	16	16	5 317	5 436
(50, False)	535.67	0.0549	1 715	1 761	2 219 776	2 267 427
(50, True)	6.14	0.0092	12	15	9 362	10 814
(100, False)	923.81	0.0386	1 923	1 938	5 077 935	5 105 263
(100, True)	10.41	0.0092	11	15	16 611	19 112
(200, False)	2284.97	0.0275	2 127	2 138	11 080 406	11 127 388
(200, True)	17.67	0.0092	11	15	33 211	36 212
(500, False)	2744.28	0.0202	1 837	1 840	23 810 402	23 839 337
(500, True)	39.43	0.0092	11	15	82 814	82 814

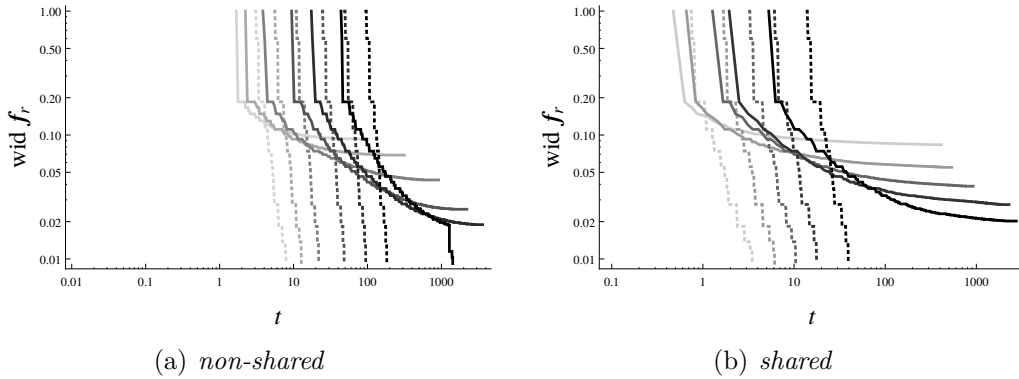


Figure 9: Problem 9: t VS $\text{wid}(\mathbf{f}_r)$

Problem 10

Table 20: Results for *non-shared* variant on Problem 10

Method	t	$\text{wid}(\mathbf{f}_r)$	$ \mathcal{S}_T $	$\max \mathcal{S}_T $	$ \mathcal{H}_\square $	$\max \mathcal{H}_\square $
(500, False)	123.25	0.2230	1 701	1 701	0	501
(500, True)	10.03	0.0072	20	22	0	501
(1 000, False)	362.96	0.1918	1 586	1 586	0	1 001
(1 000, True)	14.76	0.0072	16	21	0	1 001
(2 000, False)	1269.62	0.0841	1 508	1 509	0	2 001
(2 000, True)	25.62	0.0072	16	20	0	2 001
(5 000, False)	2985.32	0.0336	1 584	1 585	0	5 001
(5 000, True)	55.51	0.0072	16	20	0	5 001
(10 000, False)	3600.04	0.0176	1 165	1 166	0	10 001
(10 000, True)	106.30	0.0072	16	19	0	10 001
(20 000, False)	3600.49	0.0108	436	489	0	20 001
(20 000, True)	201.83	0.0072	16	19	0	20 001

Table 21: Results for *shared* variant on Problem 10

Method	t	$\text{wid}(\mathbf{f}_r)$	$ \mathcal{S}_T $	$\max \mathcal{S}_T $	$ \mathcal{H}_\square $	$\max \mathcal{H}_\square $
(20, False)	472.28	0.2104	2 207	2 310	1 131 632	1 185 399
(20, True)	5.21	0.0096	23	29	5 803	8 029
(50, False)	1014.39	0.1711	2 128	2 128	2 919 105	2 919 105
(50, True)	7.31	0.0072	20	22	11 970	12 921
(100, False)	2110.05	0.0771	1 947	1 948	5 493 969	5 496 818
(100, True)	11.14	0.0072	17	21	19 217	19 417
(200, False)	3600.03	0.0454	1 041	1 073	5 627 299	5 661 153
(200, True)	19.42	0.0072	16	20	34 616	38 017
(500, False)	3600.09	0.0217	724	762	9 216 821	9 867 993
(500, True)	43.12	0.0072	16	19	86 516	94 517

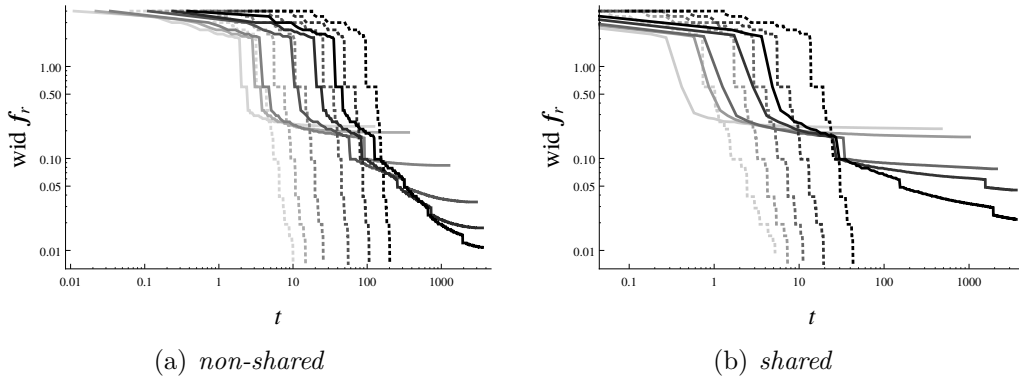


Figure 10: Problem 10: t VS $\text{wid}(\mathbf{f}_r)$